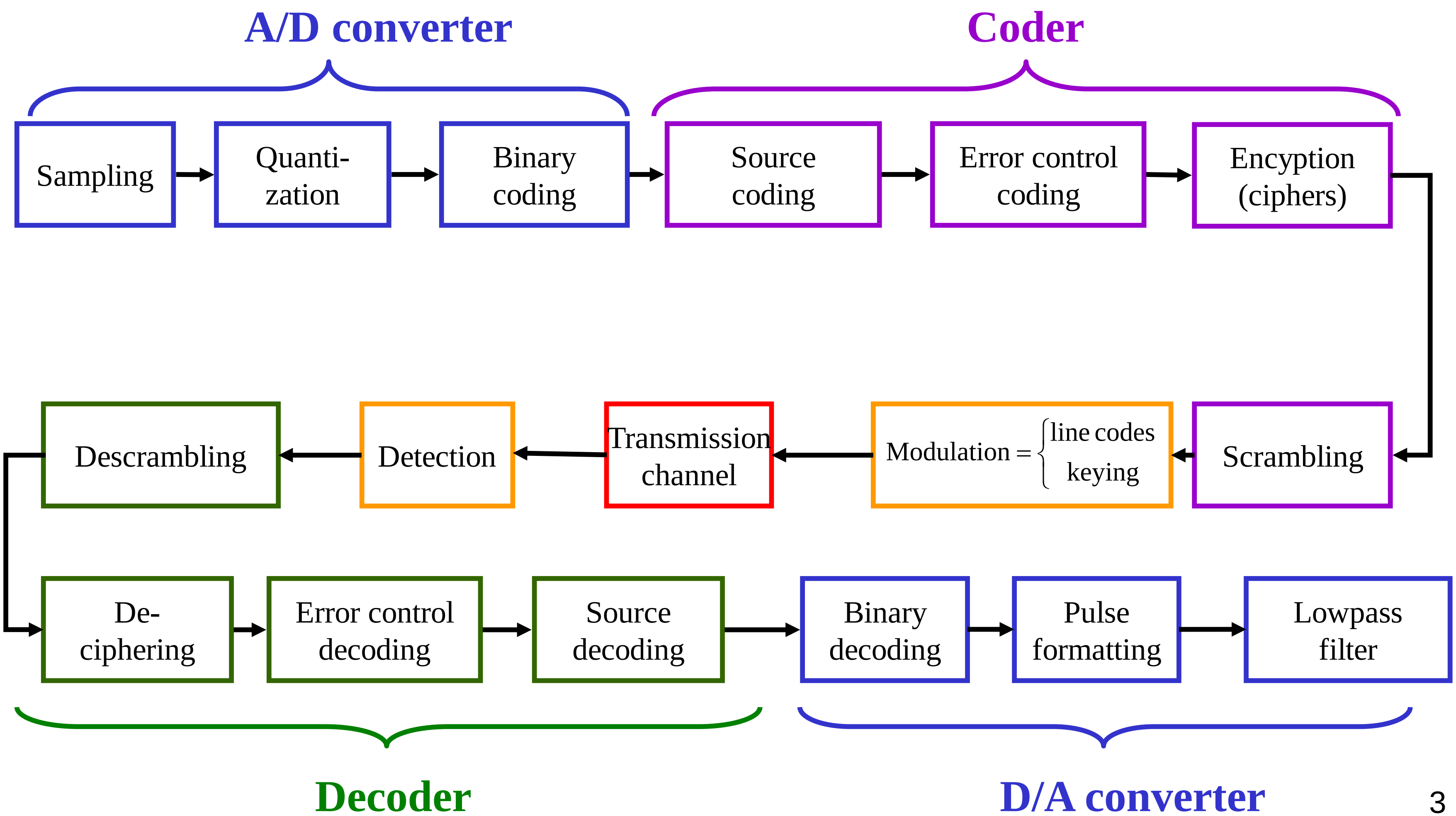
TRANSMISSION CODES (LINE CODES) (12)

Agenda

- Digital telecommunication system
- Natural digital signal (unipolar code)
- Properties of transmission codes
- Codes: bipolar, biphase, Miller's
- Differential encoding
- Pseudoternary codes: AMI & HDB, PRS
- Summary

DIGITAL TELE-COMMUNICATION SYSTEM



A/D converter

• Information signal is encoded in a binary format in a **A/D converter**.

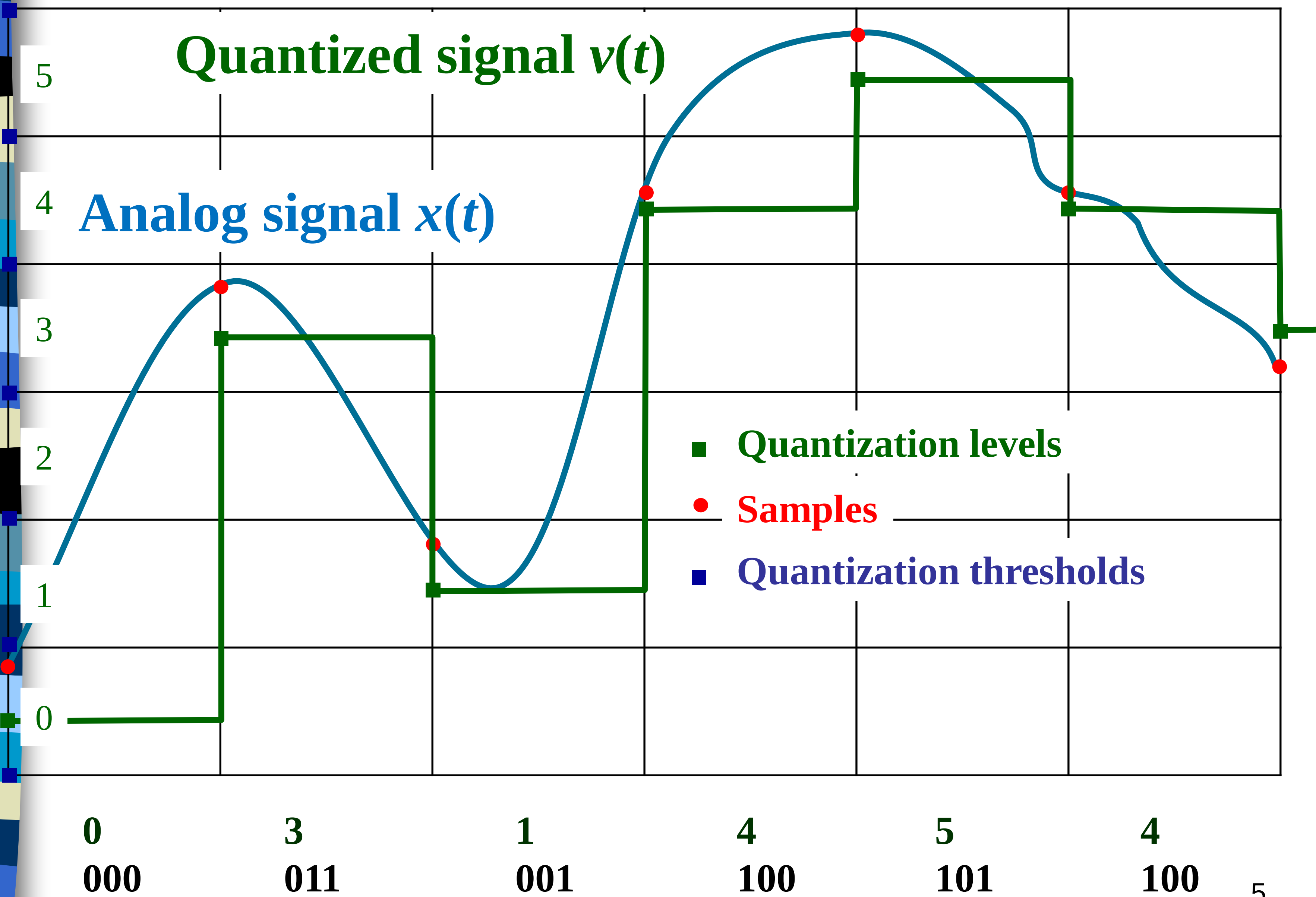
• Common solution is **sampling combined with quantization**, and then **binary coding** of decimal quantization levels.

• **Sampling** at a proper sampling rate **does not result in information loss**.

• **Quantization** does result in **irrertrivable information loss** (quantization noise). Information loss due to quantization is mitigated by increasing the numer of quantization levels.

• Transmission_rate = sampling frequency x codeword_length

Signal quantization (8 levels)



DESIRABLE PROPERTIES OF TRANSMISSION CODES

Line coding (transmission codes) maps the digital data stream into electrical symbols which are suitable for transmission over a baseband physical channel.

Several properties of a transmission code can be deduced by analyzing its power spectrum.

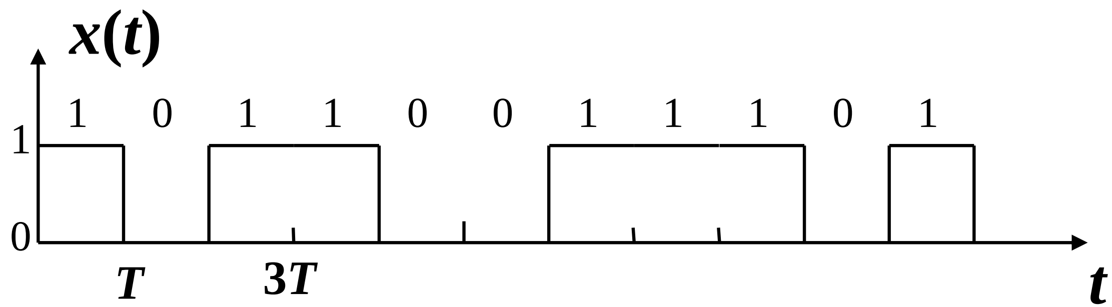
1. No dc [$\delta(\omega = 0) = 0$]; dc component is a waste of transmitter power.

2. Low level power spectrum close to $\omega = 0$ as low frequencies are strongly attenuated due to capacity properties of transmission lines.

3. Narrow bandwidth

4. Considerable amount of *timing content* needed for synchronization between transmitter and receiver.

Natural digital signal (unipolar line code)



Assumptions:

- **Bit duration (clock) *T***
- **Signalization rate $\omega_T = 2\pi/T$, $f_T = 1/T$ [bps]**
- **Binary signal {0, 1} is a sequence of independent random variables distributed as:**

$$\{(0,q),\ (1,p)\} = \{(0,1\text{-}p),(1,p)\}$$

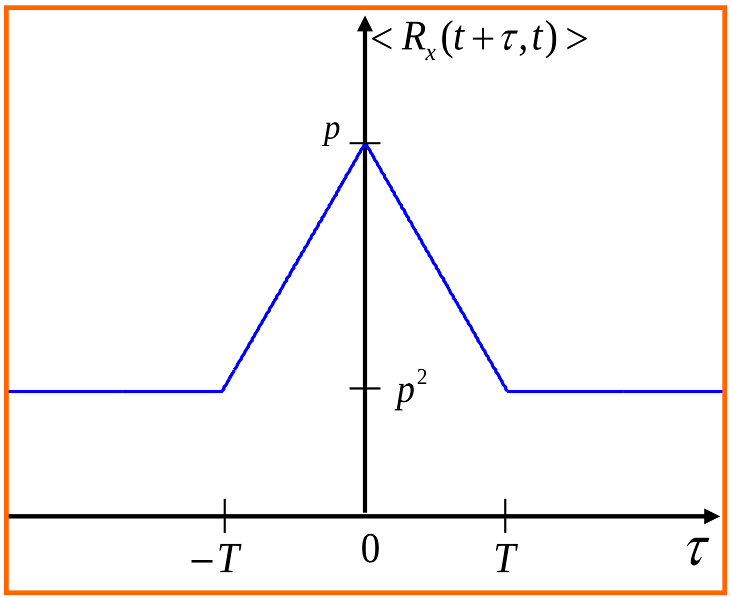
Spectrum analysis (unipolar code)

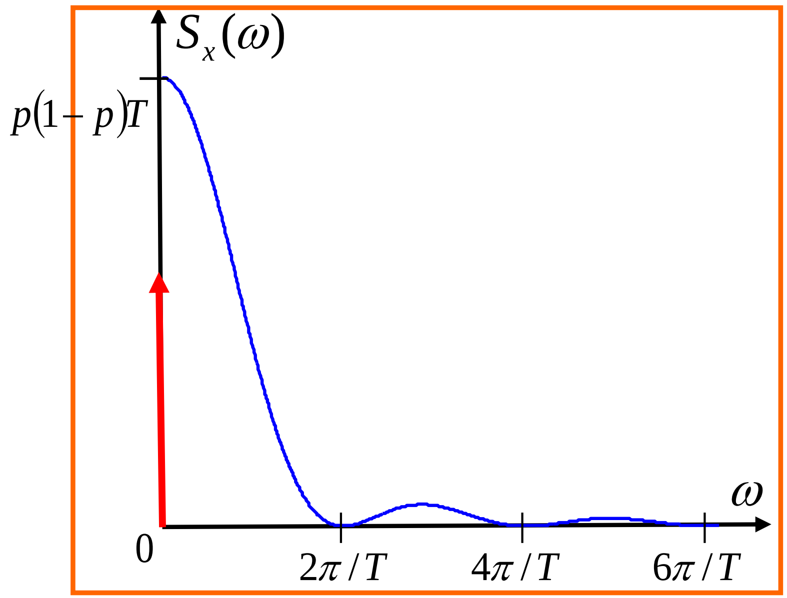
Autocorrelation function (time averaged):

$$< R_x(t+\tau,t) >= p^2 + p(1-p)\Lambda_{2T}(\tau)$$

Power spectrum:

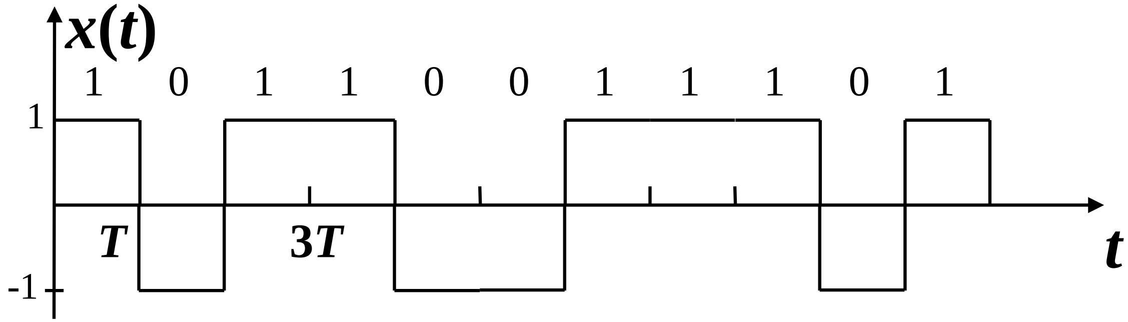
$$S_x(\omega) = \mathcal{F}\{< R_x(t+\tau,t) >\} >= 2\pi p^2 \delta(\omega) + p(1-p)T\mathrm{Sa}^2\frac{\omega T}{2}$$





Neither property 1…4 is met by the unipolar code.

Bipolar transmission code



Assumptions:

- **Bit duration (clock) *T***
- **Signalization rate $\omega_T = 2\pi/T$, $f_T = 1/T$ [bps]**
- **Binary signal {0, 1} is a sequence of independent random variables distributed as:**

$$\{(-1,q),\ (+1,p)\} = \{(-1,1\text{-}p),(+1,p)\}$$

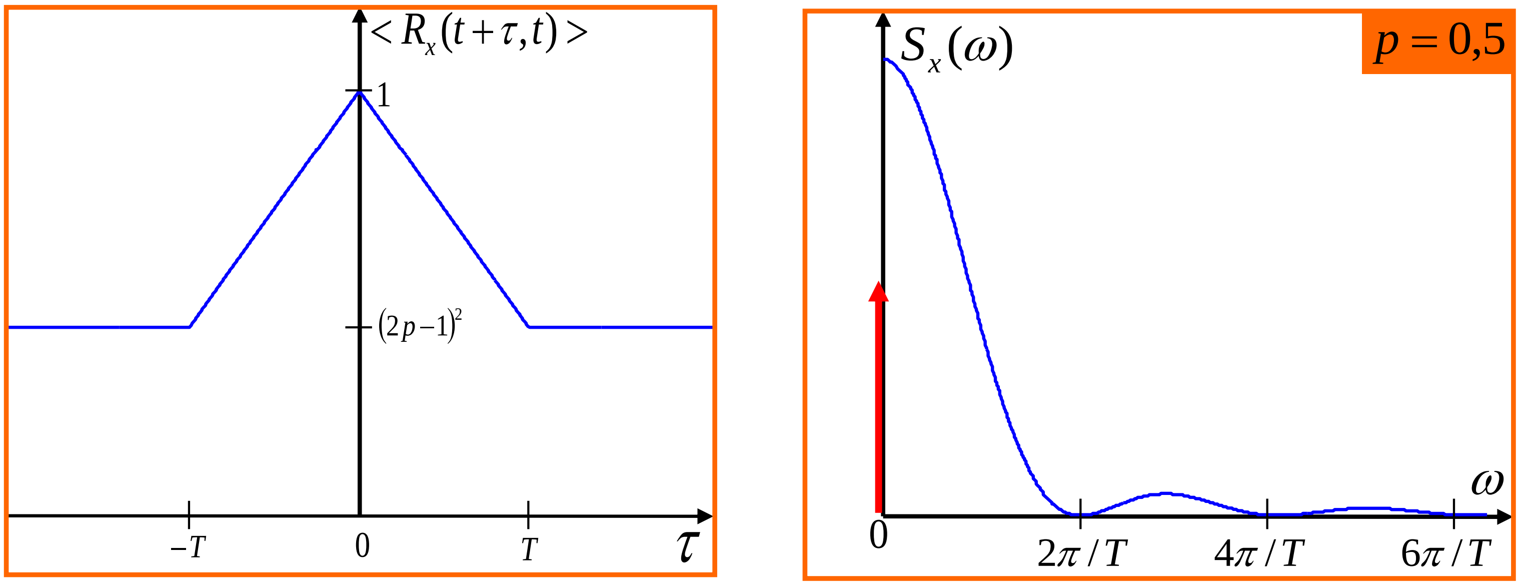
Spectral analysis (bipolar code)

Autocorrelation function (time averaged):

$$< R_x(t+\tau,t) > = (2p-1)^2 + 4p(1-p)\Lambda_{2T}(\tau)$$

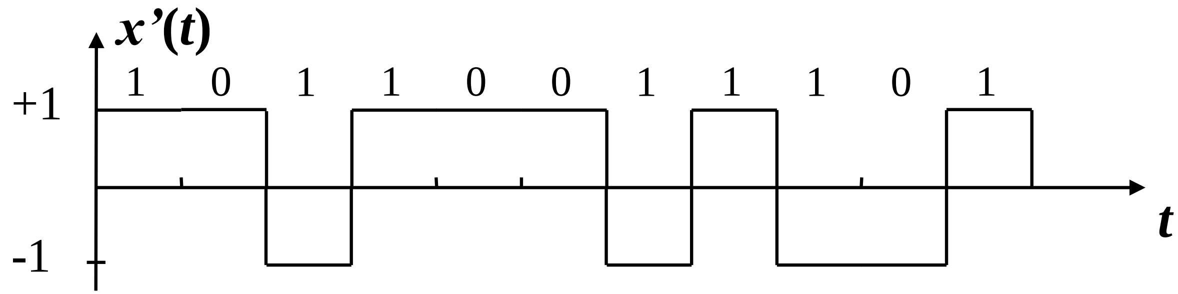
Power spectrum:

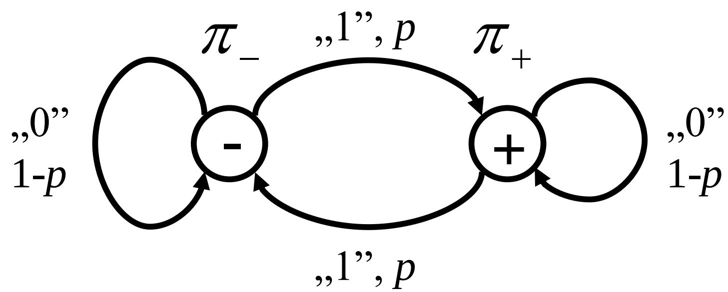
$$S_x(\omega) = 2\pi(2p-1)^2\delta(\omega) + 4p(1-p)T\mathrm{Sa}^2\frac{\omega T}{2}$$



Neither property 1…4 is met by bipolar code, though at *p* = ½ the dc content disappears.

Differential precoding





$p = \Pr\{1\}$

$1 - p = \Pr\{0\}$

$$\begin{cases} \pi_+ = \pi_+ \cdot (1-p) + \pi_- \cdot p \\ \pi_- = \pi_- \cdot (1-p) + \pi_+ \cdot p \\ \pi_+ + \pi_- = 1 \end{cases}$$

$$\pi_+ = \pi_- = \frac{1}{2}$$

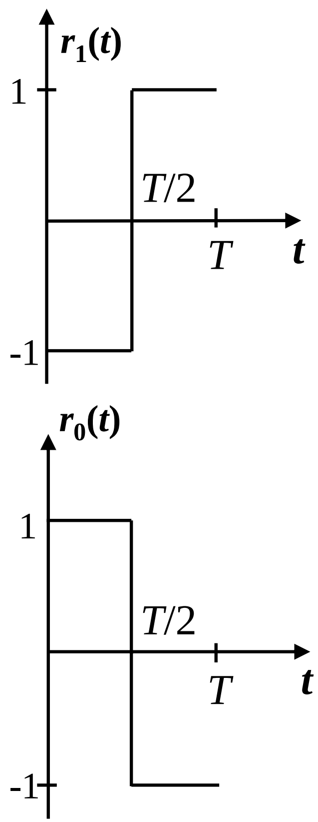
Differential precoding equalizes probability distribution thereby removing dc component.

Biphase line code

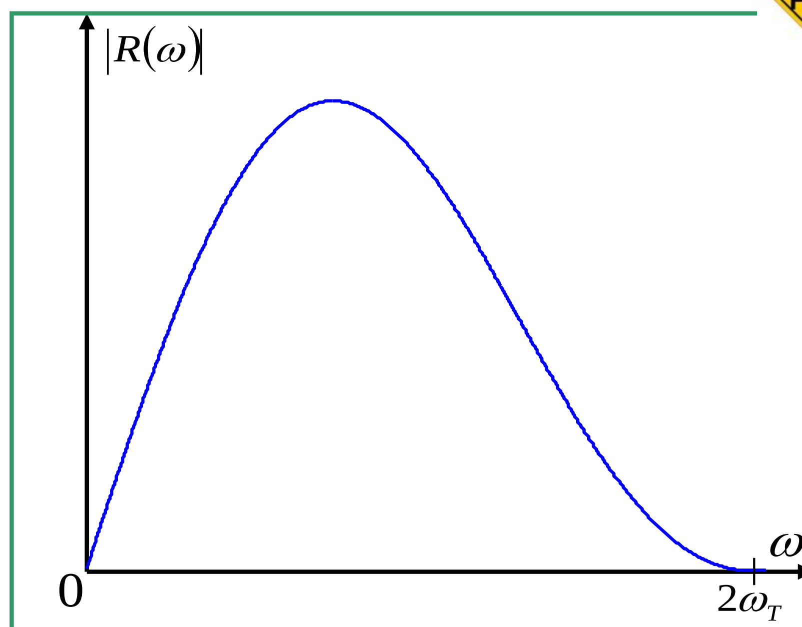
Remark: **power spectrum of a transmission code is proportional to squared a-f spectrum of a pulse.**

$$S_x(\omega) = T\mathrm{Sa}^2\frac{\omega T}{2} \sim \left|\mathrm{Sa}\frac{\omega T}{2}\right|^2 \qquad \Pi_T(t) \leftrightarrow T\mathrm{Sa}\frac{\omega T}{2}$$

Bipulse

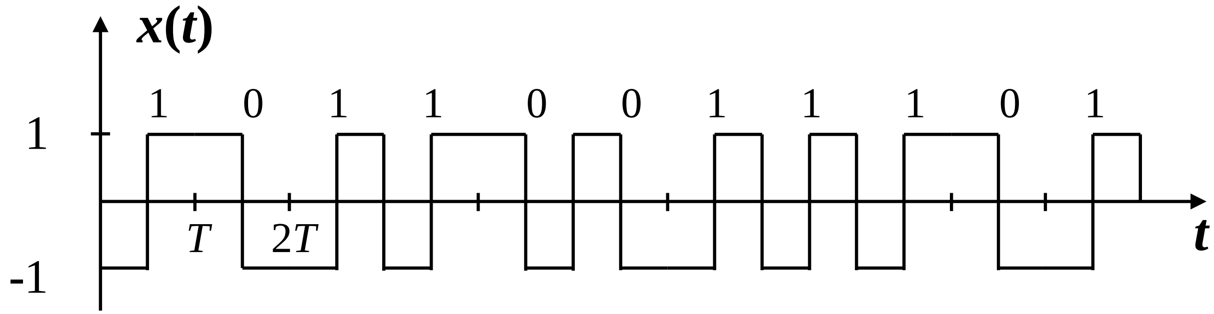


$$r(t) \leftrightarrow |R(\omega)| = T\frac{\sin^2\frac{\omega T}{4}}{\left|\frac{\omega T}{4}\right|}$$



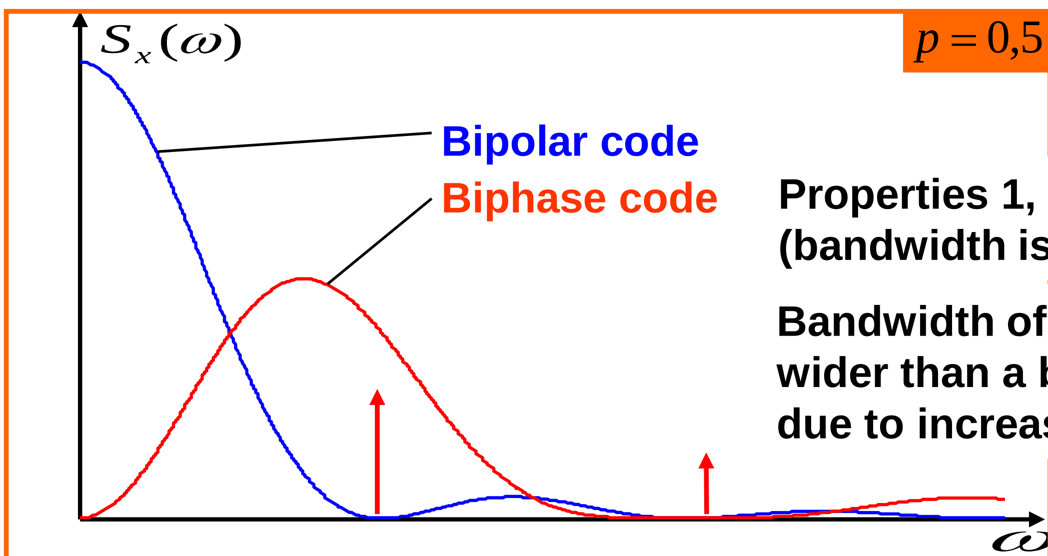
WORK IN PROGRESS

Spectrum analysis (biphase code)



Power spectrum:

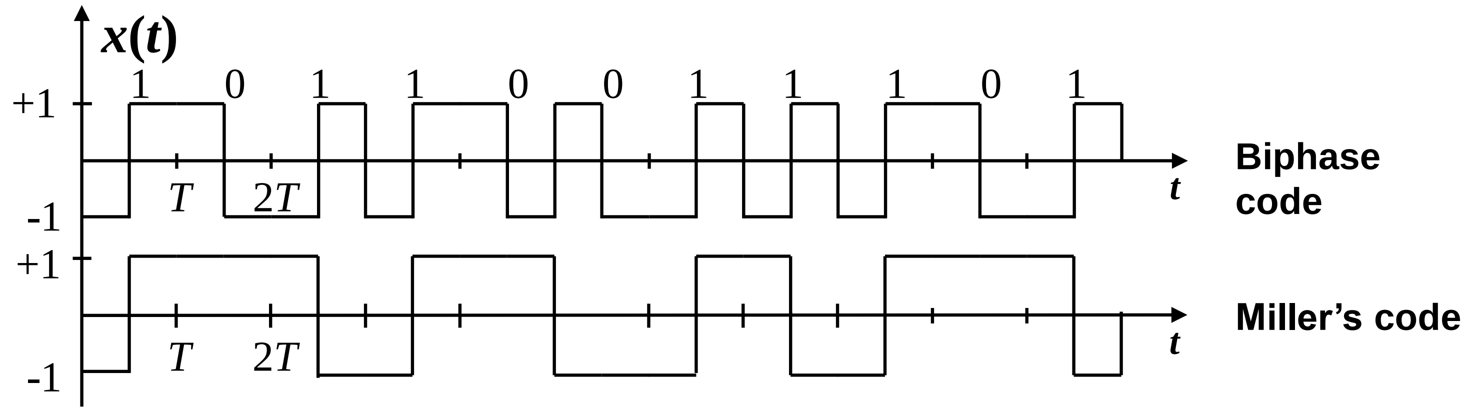
$$S_x(\omega) = \frac{8}{\pi}(2p-1)^2 \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{n^2} \delta(\omega - n\omega_T) + 4p(1-p)T \frac{\sin^4 \frac{\omega T}{4}}{\left(\frac{\omega T}{4}\right)^2}$$

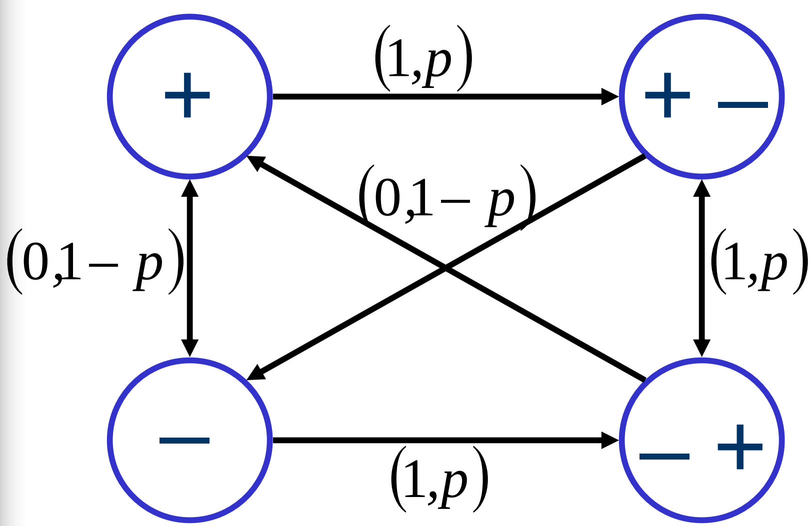


Properties 1, 2, and 4 are met; property 3 (bandwidth is not met).

Bandwidth of a biphase code spectrum is 2x wider than a bandwith of a bipolar code due to increased timing content.

Miller's line code





$$\{(0,1\text{-}p),(1,p)\}$$

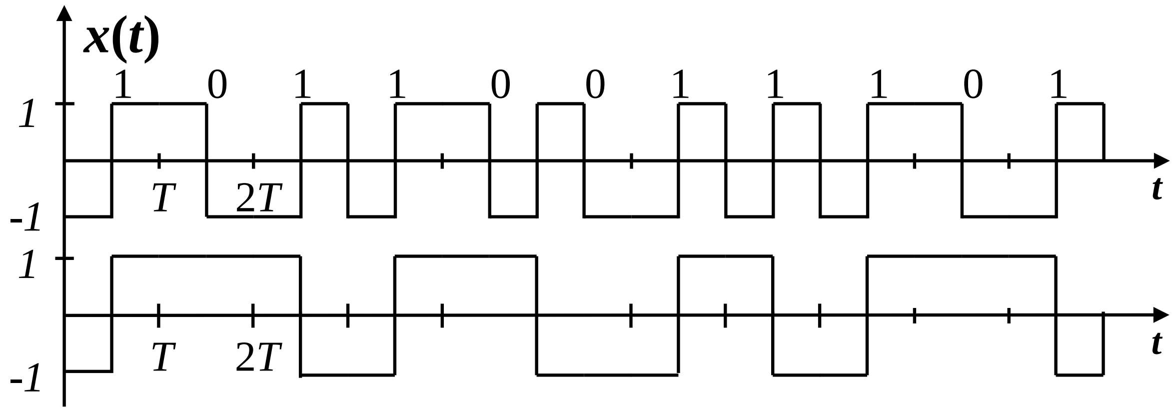
Miller's line code:

- **'0' – full pulses**
- **'1' – bipulses**
- **signal level unchanged when 1↔ 0**
- **signal level altered when 0 ↔ 0**

Explain the Miller code graph and solve it for *p* = ½.

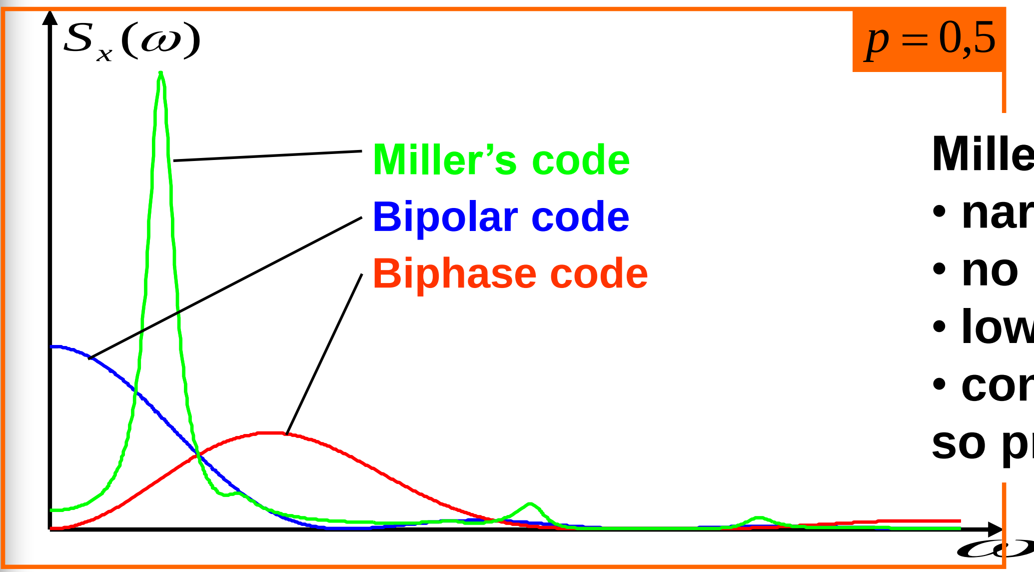
WORK IN PROGRESS

Miller's line code



Power spectrum ($\beta = \omega T/2$):

$$S_x(\omega) = \frac{T}{2\beta^2(17+8\cos 8\beta)}(23 - 2\cos\beta - 22\cos 2\beta - 12\cos 3\beta + 5\cos 4\beta + 12\cos 5\beta + 2\cos 6\beta - 8\cos 7\beta + 2\cos 8\beta)$$

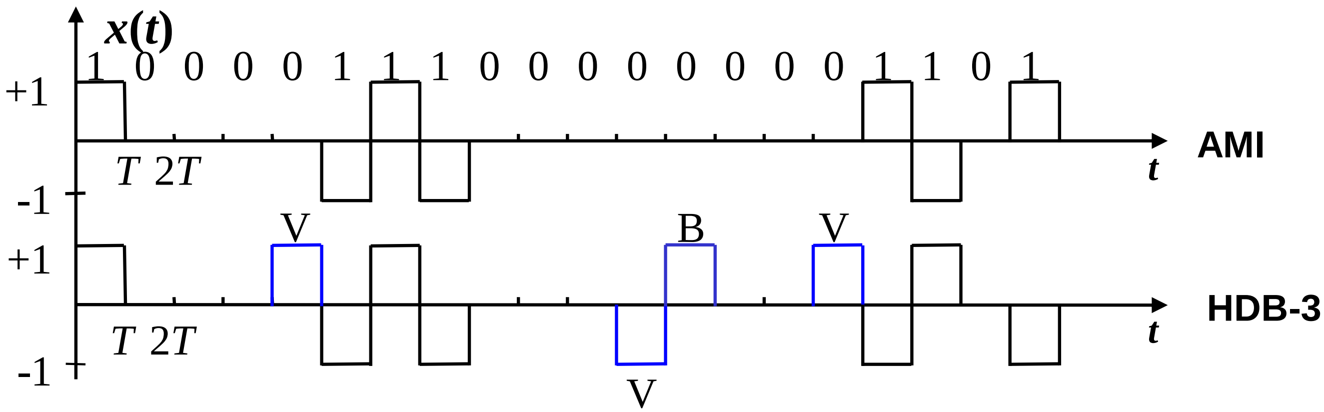


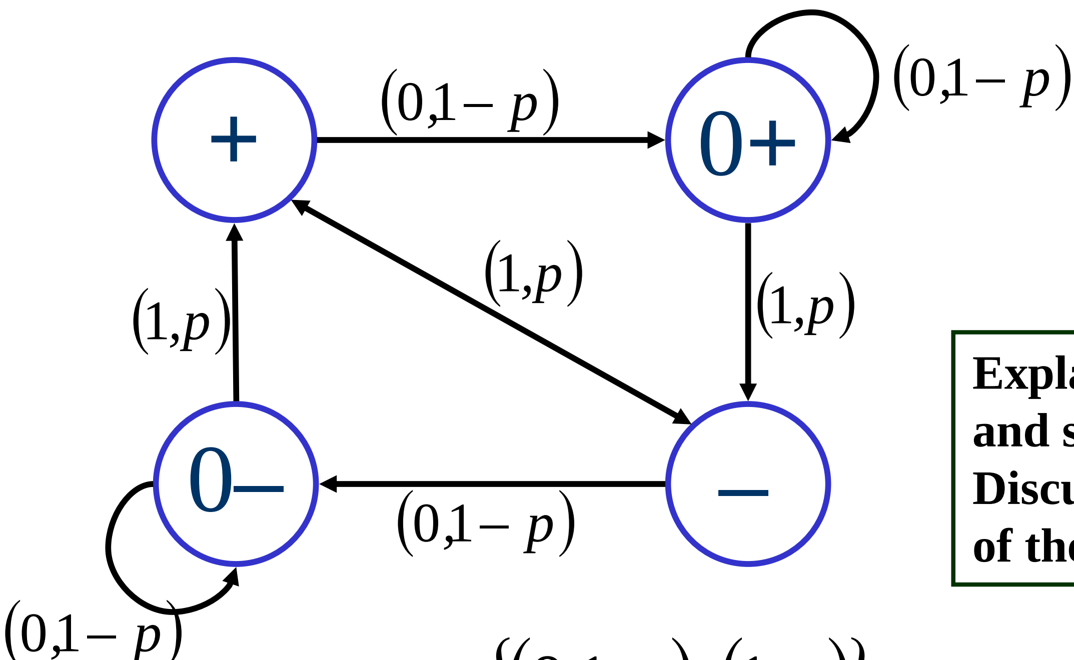
Miller's code has:

- **narrow bandwidth,**
- **no dc component,**
- **low level of low frequencies,**
- **considerable timing content**

so properties 1…4 are met.

Line code AMI (Alternate Mark Inversion)





$$\{(0,1\text{-}p),(1,p)\}$$

Line code AMI:

- **'0' – zero level**
- **'1' – impulses ±1**
- **alternating level at '1'**

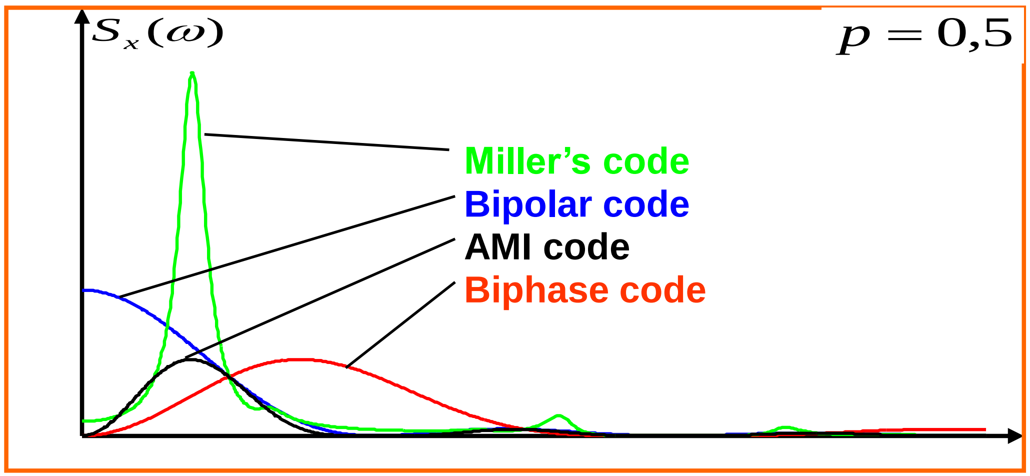
Explain the graph of the AMI code and solve it for $p = ½$.
Discuss rules of detection of the HDB-3 code.

WORK IN PROGRESS

Line code AMI (Alternate Mark Inversion)

Power spectrum:

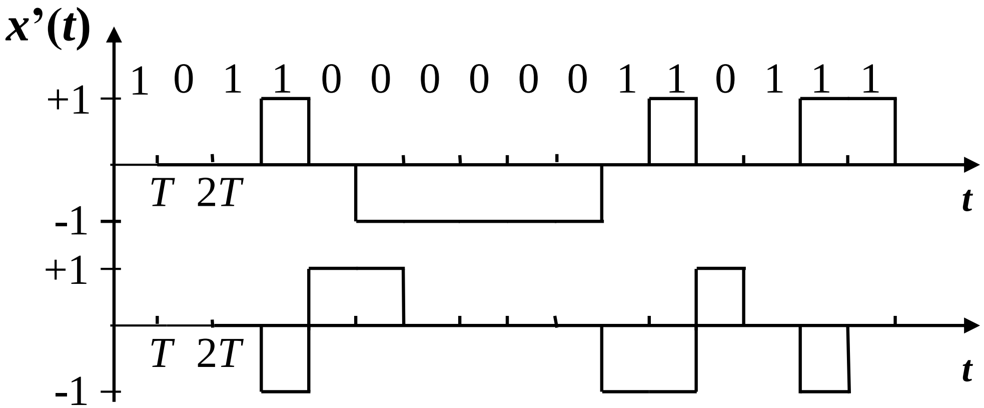
$$S_x(\omega)=4p(1-p)A_0^2T\,\frac{\sin^2\frac{\omega T}{2}\,\mathrm{Sa}^2\frac{\omega T}{2}}{1+2(2p-1)\cos\omega T+(2p-1)^2}$$



Bandwidth of AMI code is almost the same as for a bipolar code with low frequencies attenuated (properties 1, 2, and 4 are met).

More timing content is inbuilt in the HDB-3 (High Density Bipolar) code as sequences of 4 zeros („0000") are eliminated while preserving error resilience.

Partial Response Signaling (PRS)



***x*(*t*) – bipolar code**

PRS duobinarny

$$x'(t)=\frac{1}{2}\left[x(t)+x(t-T)\right]$$

PRS duobinary modified

$$x'(t)=\frac{1}{2}\left[x(t)-x(t-2T)\right]$$

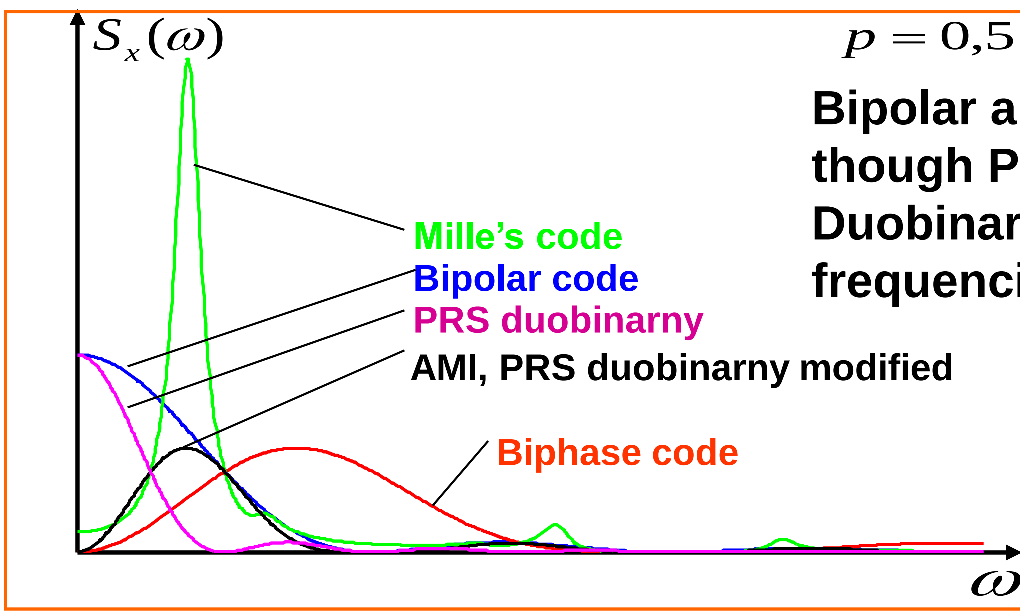
Power spectra (*p* = 0.5):

PRS duobinarny

$$S_x(\omega)=T\mathrm{Sa}^2\omega T$$

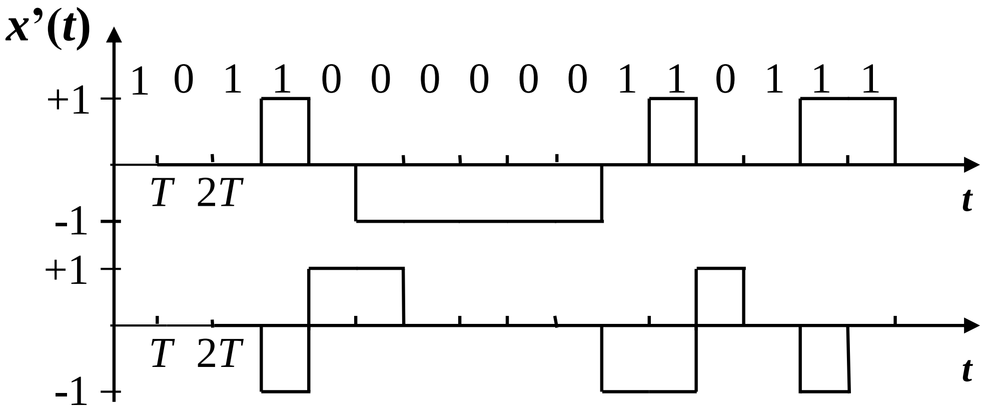
PRS duobinary modified

$$S_x(\omega)=T\sin^2\frac{\omega T}{2}\mathrm{Sa}^2\frac{\omega T}{2}$$



Bipolar and PRS codes use same pulses though PRS bandwidth is twice narrower. Duobinary PRS code does not contain low frequencies.

Partial Response Signaling (PRS)



$x(t)$ – bipolar code

PRS duobinarny

$$x'(t)=\frac{1}{2}\left[x(t)+x(t-T)\right]$$

PRS duobinary modified

$$x'(t)=\frac{1}{2}\left[x(t)-x(t-2T)\right]$$

$$x'(t)=\frac{1}{2}\left[x(t)+x(t-T)\right]=\frac{1}{2}x(t)*\left[\delta(t)+\delta(t-T)\right]$$

Determine the spectrum of the PRS duobinary code.

$$x(t) \longrightarrow \boxed{\begin{array}{l}\delta(t)+\delta(t-T)\leftrightarrow \\ \leftrightarrow 1+\exp(-j\omega T)\end{array}} \longrightarrow x'(t)$$

$$S_{x'}(\omega)=\frac{1}{4}S_x(\omega)\times\left|1+\exp(-j\omega T)\right|^2$$

$$S_{x'}(\omega)=T\mathrm{Sa}^2\omega T$$

SUMMARY

- Transmission code is a mapping of a binary signal into an electrical signal.

- Desired properties of each transmission code: narrow bandwidth, no dc component, low level of low frequencies, and considerable timing content.

- Properties of a transmission code may be assessed by its spectral analysis.

- Properties of a transmission code may be tuned either by and impulse shape and/or coding rule.

- Differential precoding eliminates dc component.